FURTHER SUPPLEMENT FOR BAYESIAN INFERENCE IN A CLASS OF PARTIALLY IDENTIFIED MODELS

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This further supplement contains even further additional material. Section 11.1 discusses misspecification, and section 11.2 discusses further empirical results.

11.1. **Misspecification.** A common concern in applied work is the behavior of an estimator under model misspecification. Many estimators in point identified models have the feature that even if the model is misspecified, the estimator still estimates a useful parameter.[1] This section briefly describes the "expected" behavior under misspecification in the context of an interval identified parameter.

It is useful to consider the behavior of posterior probability statements that condition on the identified set being non-empty, which "imposes" the assumption of correct specification even when that assumption is false. These are the posterior probabilities of the form $\Pi(\cdot|X, \Theta_I \neq \emptyset)$. As shown in the theoretical results, the posterior probabilities in a misspecified model that do not condition on the identified set being non-empty (i.e., $\Pi(\cdot|X)$) will tend in large samples toward estimating that indeed the identified set is empty, which is perhaps the desired behavior under misspecification since in that case indeed the identified set is empty. However, conditioning on $\Theta_I \neq \emptyset$ can be used as a way to get a "pseudo-true" identified set even under misspecification. This method amounts to "rejecting" draws from the posterior of $\mu|X$ that violate a certain condition (namely, non-emptiness of the identified set), which more generally has been implemented in other settings. For example, Uhlig (2005) imposes an identification constraint concerning sign restrictions on an impulse response function in VAR models by "rejecting" draws from the posterior that violate that restriction.

Essentially, based on $\Pi(\cdot|X, \Theta_I \neq \emptyset)$, the posterior should be expected to "estimate" the identified set corresponding to the value of μ that is "closest" (under a metric induced by the posterior for μ) to the true μ_0 among all values of μ that result in a non-empty identified set. In that sense, this approach estimates a "pseudo-true" identified set under misspecification. This can be most easily seen in the context of an interval identified parameter from example 1 summarized in figure 6, but the basic idea immediately generalizes to any model.

[1]Ordinary least squares regression estimates the minimum mean square error linear approximation to the conditional expectation, quantile regression estimates a certain weighted mean square error (i.e., Angrist, Chernozhukov, and Fernández-Val (2006)), and maximum likelihood estimates a "pseudo-true" parameter that minimizes the Kullback-Leibler divergence to the truth (e.g., White (1982)). The situation might be different in partially identified models (e.g., Ponomareva and Tamer (2011)).

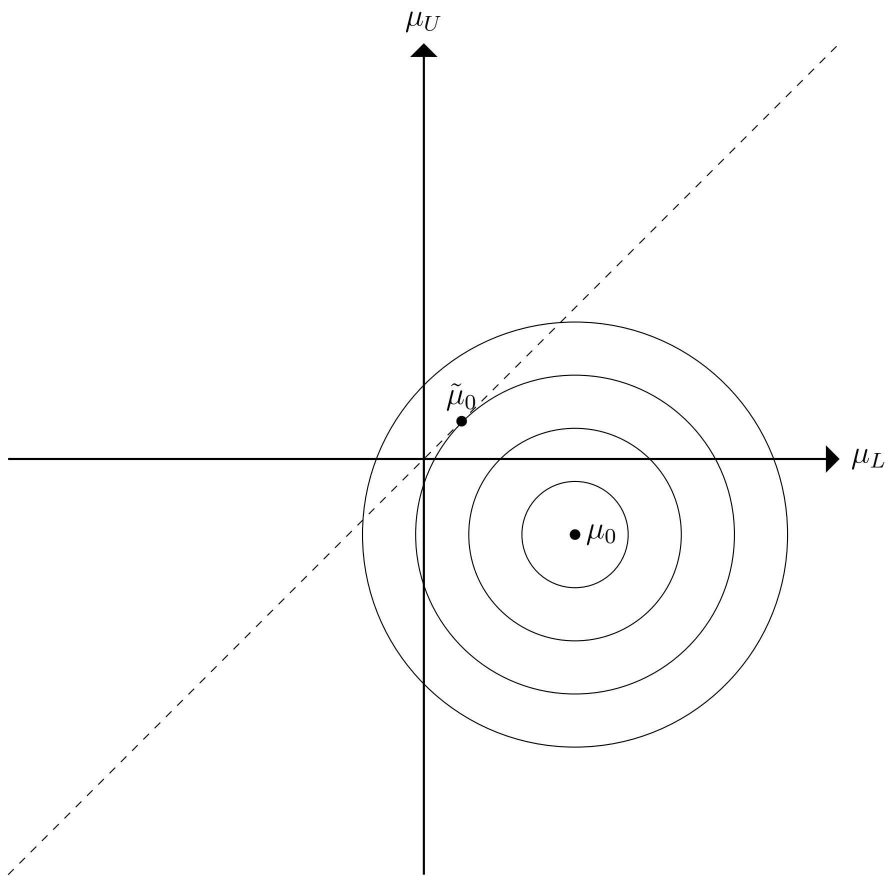


FIGURE 6. Behavior under misspecification

This figure shows μ_L along the horizontal axis, and μ_U along the vertical axis. The dashed diagonal line corresponds to the values of μ such that θ is point identified, since along that line $\mu_L = \mu_U$, so it must be that $\theta = \mu_L = \mu_U$. Values of μ above the diagonal line correspond to values of μ such that θ is partially identified, since there $\mu_L < \mu_U$. Finally, values of μ below the diagonal line correspond to value of μ such that the model is misspecified, since there $\mu_L > \mu_U$. So, suppose that the true μ_0 indeed is below the diagonal line, as reflected in the figure. The posterior for μ is unaffected by the fact that the model is misspecified, so the posterior for μ still behaves like a conventional posterior for a point identified parameter. The concentric circles emanating from μ_0 represent the level sets of the posterior for μ, with values of μ closer to μ_0 more likely according to the posterior for μ. (It is assumed here that the posterior is centered at the true μ_0, which will be approximately true at least in large samples under assumptions 2 and 3. More generally, the posterior will be centered at approximately $\mu_n(X)$ per assumption 3.)

Now consider the behavior of $\Pi(\cdot|X, \Theta_I \neq \emptyset)$. This is concerned with the posterior for μ, conditioning on the fact that the identified set is non-empty. In the interval identified parameter model, this is equivalent to conditioning on $\mu_L \leq \mu_U$. Therefore, $\Pi(\cdot|X, \Theta_I \neq \emptyset)$ is based on the posterior $\mu|(X, \mu_L \leq \mu_U)$. The most likely draws from $\mu|(X, \mu_L \leq \mu_U)$ will be in a neighborhood of $\tilde{\mu}_0$, which is the point in the set $\{\mu : \mu_L \leq$

$\mu_U\}$ that has highest density according to the posterior for $\mu|X$. As a consequence, inference on the identified set conditioning on non-emptiness of the identified set will tend to be centered around the identified set evaluated at $\tilde{\mu}_0$, and in that sense this approach estimates a "pseudo-true" identified set under misspecification. The basic idea immediately generalizes to other models.

11.2. **Further empirical results.**

11.2.1. *Model with market presence.* Another specification uses only the firm- and market-specific binary explanatory variable: *market presence*. In this specification, the payoff of firm LCC if it enters market m is

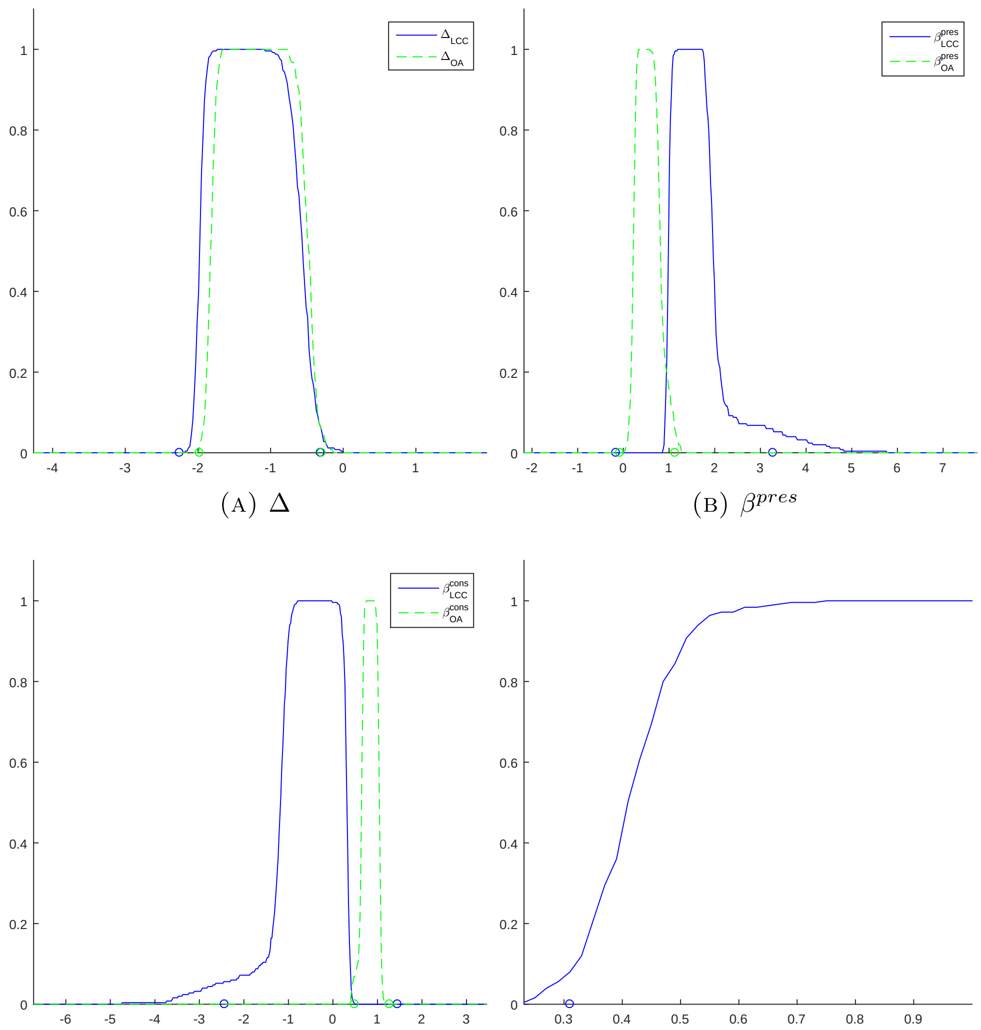
$$\beta_{LCC}^{cons} + \beta_{LCC}^{pres} X_{LCCm,pres} + \Delta_{LCC} y_{OAm} + \epsilon_{LCCm}$$

and similarly the payoff of firm OA if it enters market m is

$$\beta_{OA}^{cons} + \beta_{OA}^{pres} X_{OAm,pres} + \Delta_{OA} y_{LCCm} + \epsilon_{OAm}.$$

In this specification, θ is a 7-dimensional vector. The point identified parameter μ is a 16-dimensional vector of conditional choice probabilities: there are four types of markets (because there are two binary explanatory variables per market) and each type of market is summarized by four choice probabilities (because there are two firms each of which has a binary decision to enter or not).

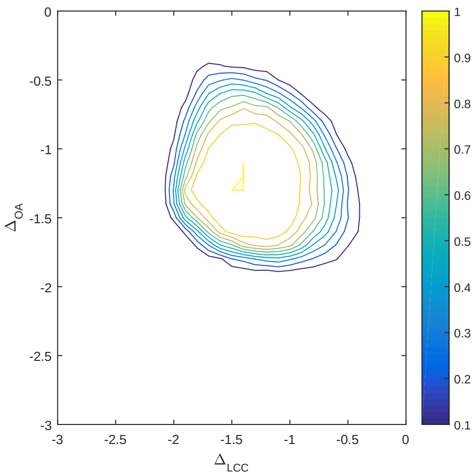
Figure 7 reports the posterior probabilities that various parameter values belong to the identified set. The posterior probabilities for the identified sets for the Δ parameters seem essentially the same across the two types of firms. The effect of market presence on the payoff of both firms is almost certainly positive, in the sense that the posterior "curve" for both firms is close to zero on negative values, and the effect of having a greater market presence seems higher for LCC firms, in the sense that the posterior "curve" for the LCC firms is relatively greater on larger values of the parameter space. The monopoly profits associated with below-median market presence (i.e., the constant term) seems lower for LCC firms. Finally, the "curve" of posterior probabilities associated with ρ is basically flat and equal to one for values of ρ greater than approximately 0.5, implying that any sufficiently high correlation almost certainly could have generated the data. The circles along the horizontal axes in figure 7 are the endpoints of the 95% credible sets for the identified set for the corresponding parameter. Figure 8 displays the posterior probabilities over the identified sets for a few pairs of parameters.



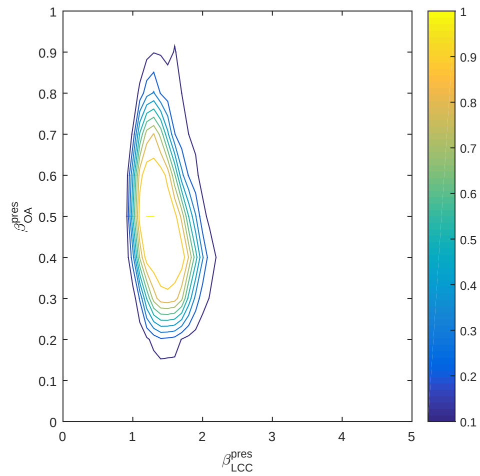
(A) Δ (B) β^{pres}

(C) β^{cons} (D) ρ

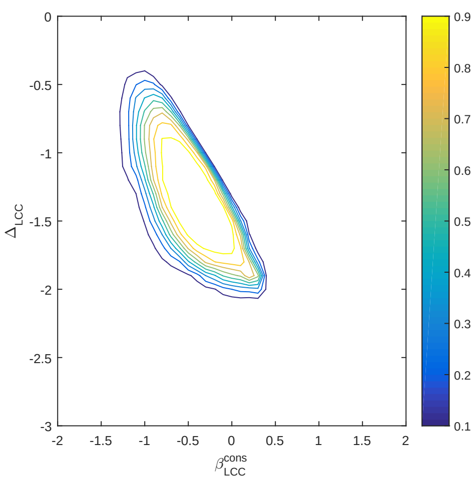
FIGURE 7. Posterior probabilities that various parameter values belong to the identified set in model with market presence only



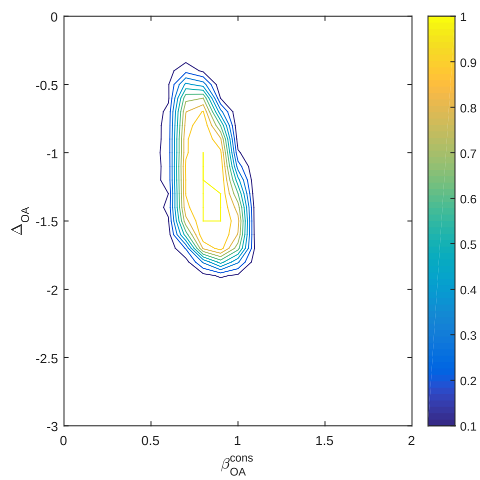
(A) $(\Delta_{LCC}, \Delta_{OA})$



(B) $(\beta^{pres}_{LCC}, \beta^{pres}_{OA})$



(C) $(\beta^{cons}_{LCC}, \Delta_{LCC})$



(D) $(\beta^{cons}_{OA}, \Delta_{OA})$

FIGURE 8. Posterior probabilities that various pairs of parameter values belong to the identified set in model with market presence only

References

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